

VISIT...

LANZAROTE
Caliente.COM

Chapter 12

THERMAL MODELING AND COOLING

12.1. INTRODUCTION

Besides electromagnetic, mechanical and thermal designs are equally important.

Thermal modeling of an electric machine is in fact more nonlinear than electromagnetic modeling. Any electric machine design is highly thermally constrained.

The heat transfer in an induction motor depends on the level and location of losses, machine geometry, and the method of cooling.

Electric machines work in environments with temperatures varied, say from -20°C to 50°C , or from 20° to 100° in special applications.

The thermal design should make sure that the motor windings temperatures do not exceed the limit for the pertinent insulation class, in the worst situation. Heat removal and the temperature distribution within the induction motor are the two major objectives of thermal design. Finding the highest winding temperature spots is crucial to insulation (and machine) working life.

The maximum winding temperatures in relation to insulation classes shown in [Table 12.1](#).

Table 12.1. Insulation classes

Insulation class	Typical winding temperature limit [$^{\circ}\text{C}$]
Class A	105
Class B	130
Class F	155
Class H	180

Practice has shown that increasing the winding temperature over the insulation class limit reduces the insulation life L versus its value L_0 at the insulation class temperature ([Figure 12.1](#)).

$$\text{Log } L \approx a + \frac{b}{T} \quad (12.1)$$

It is very important to set the maximum winding temperature as a design constraint. The highest temperature spot is usually located in the stator end connections. The rotor cage bars experience a larger temperature, but they are not, in general, insulated from the rotor core. If they are, the maximum (insulation class dependent) rotor cage temperature also has to be observed.

The thermal modeling depends essentially on the cooling approach.

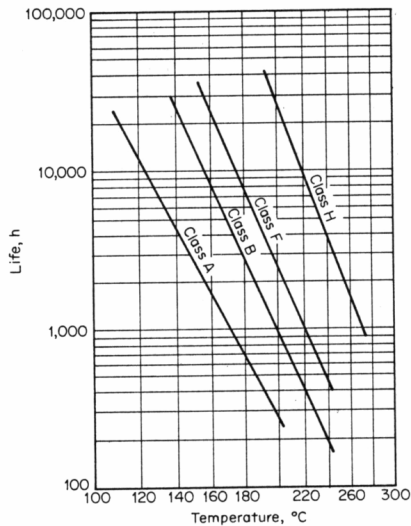


Figure 12.1 Insulation life versus temperature rise

12.2. SOME AIR COOLING METHODS FOR IMs

For induction motors, there are four main classes of cooling systems

- Totally enclosed design with natural (zero air speed) ventilation (TENV)
- Drip-proof axial internal cooling
- Drip-proof radial internal cooling
- Drip-proof radial-axial cooling

In general, fan air-cooling is typical for induction motors. Only for very large powers is a second heat exchange medium (forced air or liquid) used in the stator to transfer the heat to the ambient.

TENV induction motors are typical for special servos to be mounted on machine tools etc., where limited space is available. It is also common for some static power converter-fed IMs, that operate at large loads for extended periods of time at low speeds to have an external ventilator running at constant speed to maintain high cooling in all conditions.

The totally enclosed motor cooling system with external ventilator only (Figure 12.2b) has been extended lately to hundreds of kW by using finned stator frames.

Radial and radial-axial cooling systems (Figure 12.2c, d) are in favor for medium and large powers.

However, axial cooling with internal ventilator and rotor, stator axial channels in the core, and special rotor slots seem to gain ground for very large power as it allows lower rotor diameter and, finally, greater efficiency is obtained, especially with two pole motors (Figure 12.3). [2]

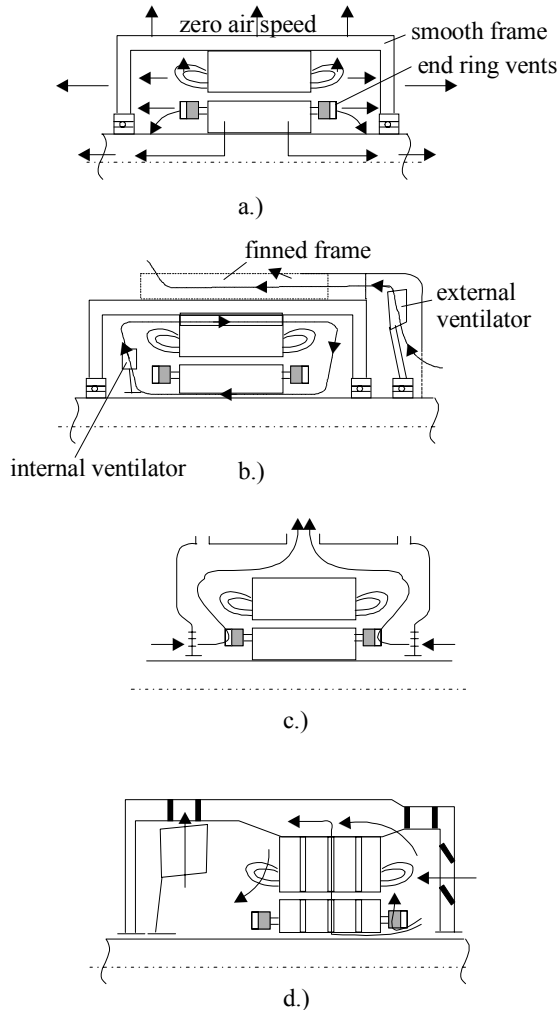


Figure 12.2 Cooling methods for induction machines
a.) totally enclosed naturally ventilated (TENV);
b.) totally enclosed motor with internal and external ventilator
c.) radially cooled IM d.) radial – axial cooling system

The rotor slots are provided with axial channels to facilitate a kind of direct cooling.

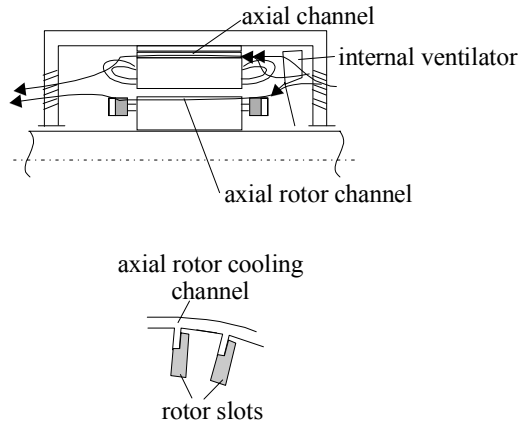


Figure. 12.3 Axial cooling of large IMs

The rather complex (anisotropic) structure of the IM for all cooling systems presented in [Figures 12.2](#) and [12.3](#) suggests that the thermal modeling has to be rather difficult to build.

There are thermal circuit models and distributed (FEM) models. Thermal circuit models are similar to electric circuits and they may be used both for thermal steady state and transients. They are less precise but easy to handle and require a smaller computation effort. In contrast, distributed (FEM) models are more precise but require large amounts of computation time.

We will define first the elements of thermal circuits based on the three basic methods of heat transfer: conduction, convection and radiation.

12.3. CONDUCTION HEAT TRANSFER

Heat transfer is related to thermal energy flow from a heat source to a heat sink.

In electric (induction) machines, the thermal energy flows from the windings in slots to laminated core teeth through the conductor insulation and slot line insulation.

On the other hand, part of the thermal energy in the end-connection windings is transferred through thermal conduction through the conductors axially toward the winding part in slots. A similar heat flow through thermal conduction takes place in the rotor cage and end rings.

There is also thermal conduction from the stator core to the frame through the back core iron region and from rotor cage to rotor core, respectively, to shaft and axially along the shaft. Part of the conduction heat now flows through the slot insulation to core to be directed axially through the laminated core. The presence of lamination insulation layers will make the thermal conduction along the axial direction more difficult. In long stack IMs, axial temperature differentials of a few degrees (less than 10^0C in general), ([Figure 12.4](#)), occur.

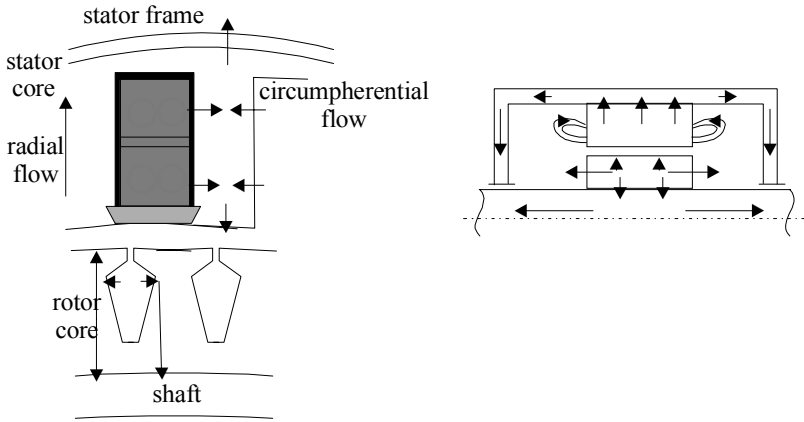


Figure 12.4 Heat conduction flow routs in the IM

So, to a first approximation, the axial heat flow may be neglected.

Second, after accounting for conduction heat flow from windings in slots to the core teeth, the machine circumferential symmetry makes possible the neglecting of circumferential temperature variation.

So we end up with a one-dimensional temperature variation, along the radial direction. For this crude approximation defining thermal conduction, convection, and radiation, and of the equivalent circuit becomes a rather simple task.

The Fourier's law of conduction may be written, for steady state, as

$$\nabla(-K\Delta\theta) = q \quad (12.2)$$

where q is heat generation rate per unit volume (W/m^3); K is thermal conductivity ($\text{W}/\text{m}, ^\circ\text{C}$) and θ is local temperature.

For one-dimensional heat conduction, Equation (12.2), with constant thermal conductivity K , becomes:

$$-K \frac{\partial^2 \theta}{\partial x^2} = q \quad (12.3)$$

A basic heat conduction element (Figure 12.5) shows that power Q transported along distance l of cross section A is

$$Q \approx q \cdot l \cdot A \quad (12.4)$$

with q , A – constant along distance l .

The thermal conduction resistance R_{con} may be defined as similar to electrical resistance.

$$R_{\text{con}} = \frac{1}{KA} \left[^\circ\text{C}/\text{W} \right] \quad (12.5)$$

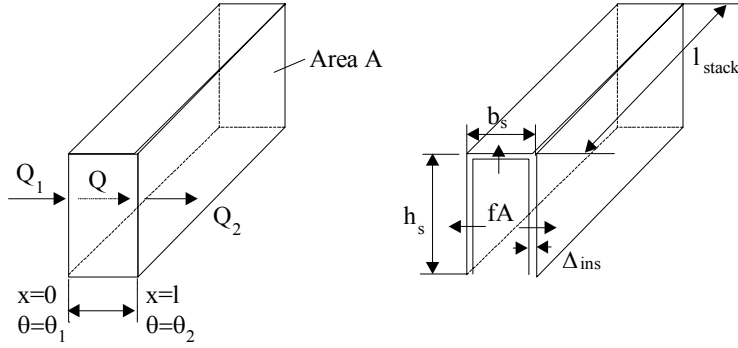


Figure 12.5 One dimensional heat conduction

Temperature takes the place of voltage and power (losses) replaces the electrical current.

For a short l , the Fourier's law in differential form yields

$$f \approx -K \frac{\Delta\theta}{\Delta x}; \quad f - \text{heat flow density} \left[\text{W/m}^2 \right] \quad (12.6)$$

If the heat source is in a thin layer,

$$f = \frac{p_{\cos}}{A} \quad (12.7)$$

$p_{\cos}$ in watts is the electric power producing losses and A the cross-section area.

For the heat conduction through slot insulation Δ_{ins} (total, including all conductor insulation layers from the slot middle (Figure 12.5)), the conduction area A is

$$A = (2h_s + b_s)l_{\text{stack}}N_s; \quad N_s - \text{slots/stator} \quad (12.8)$$

The temperature differential between winding in slots and the core teeth $\Delta\theta_{\text{Co}}$ is

$$\Delta\theta_{\text{Cos}} = p_{\cos}R_{\text{con}}; \quad R_{\text{con}} = \frac{\Delta_{\text{ins}}}{AK} \quad (12.9)$$

In well-designed IMs, $\Delta\theta_{\text{cos}} < 10^\circ\text{C}$ with notably smaller values for small power induction motors.

The improvement of insulation materials in terms of thermal conductivity and in thickness reduction have been decisive factors in reducing the slot insulation conductor temperature differential. Thermal conductivity varies with temperature and is constant only to a first approximation. Typical values are given in Table 12.2. The low axial thermal conductivity of the laminated cores is evident.

Table 12.2. Thermal conductivity

Material	Thermal conductivity (W/m⁰C)	Specific heat coefficient C_s (J/Kg⁰C)
Copper	383	380
Aluminum	204	900
Carbonsteel	45	
Motor grade steel	23	500
Si steel lamination		490
– Radial;	20 – 30	
– Axial	2.0	
Micasheet	0.43	-
Varnished cambric	2.0	-
Press board Normex	0.13	-

12.4. CONVECTION HEAT TRANSFER

Convection heat transfer takes place between the surface of a solid body (the stator frame) and a fluid (air, for example) by the movement of the fluid.

The temperature of a fluid (air) in contact with a hotter solid body rises and sets a fluid circulation and thus heat transfer to the fluid occurs.

The heat flow out of a body by convection is

$$q_{\text{conv}} = hA\Delta\theta \quad (12.10)$$

where A is the solid body area in contact with the fluid; $\Delta\theta$ is the temperature differential between the solid body and bulk of the fluid, and h is the convection heat coefficient (W/m².⁰C).

The convection heat transfer coefficient depends on the velocity of the fluid, fluid properties (viscosity, density, thermal conductivity), the solid body geometry, and orientation. For free convection (zero forced air speed and smooth solid body surface [2])

$$\begin{aligned} h_{\text{Co}} &\approx 2.158(\Delta\theta)^{0.25} \quad \text{vertical up - } W/(m^2C) \\ h_{\text{Co}} &\approx 0.496(\Delta\theta)^{0.25} \quad \text{vertical down - } W/(m^2C) \\ h_{\text{Co}} &\approx 0.67(\Delta\theta)^{0.25} \quad \text{horizontal - } W/(m^2C) \end{aligned} \quad (12.11)$$

where $\Delta\theta$ is the temperature differential between the solid body and the fluid.

For $\Delta\theta = 20^\circ\text{C}$ (stator frame $\theta_1 = 60^\circ\text{C}$, ambient temperature $\theta_2 = 40^\circ\text{C}$) and vertical – up surface

$$h_{\text{Co}} = 2.158(60 - 40)^{0.25} = 4.5 W/(m^2C)$$

When air is blown with a speed U along the solid surfaces, the convection heat transfer coefficient h_c is

$$h_c^0(u) = h_{\text{Co}}(1 + K\sqrt{U}) \quad (12.12)$$

with $K = 1.3$ for perfect air blown surface; $K = 1.0$ for the winding end connection surface, $K = 0.8$ for the active surface of rotor, $K = 0.5$ for the external stator frame.

Alternatively,

$$h_c(u) = 1.77 \frac{U^{0.75}}{L^{0.25}} \quad W/(m^2 \cdot ^\circ C) \quad \text{for } U > 5 \text{ m/s} \quad (12.13)$$

U in m/s and L is the length of surface in m.

For a closed air blown surface – inside the machine:

$$h_c^e(U) = h_{co} \left((1 + K\sqrt{U}) (1 - a/2) \right); \quad a = \frac{\theta_{air}}{\theta_a} \quad (12.14)$$

θ_{air} —local air heating; θ_a —heating (temperature) of solid surface.

In general, $\theta_{air} = 35 - 40^\circ C$ while θ_a varies with machine insulation class. So, in general, $a < 1$.

For convection heat transfer coefficient in axial channels of length, L (12.13) is to be used.

In radial cooling channels, $h_c^e(U)$ does not depend on the channel's length, but only on speed.

$$h_c^e(U) \approx 23.11 U^{0.75} (W / m^2 \cdot ^\circ C) \quad (12.15)$$

12.5. HEAT TRANSFER BY RADIATION

Between two bodies at different temperatures there is a heat transfer by radiation. One body radiates heat and the other absorbs heat. Bodies which do not reflect heat, but absorb it, are called black bodies.

Energy radiated from a body with emissivity ε to black surroundings is

$$q_{rad} = \sigma \varepsilon A (\theta_1^4 - \theta_2^4) = \sigma \varepsilon A (\theta_1 + \theta_2) (\theta_1^2 + \theta_2^2) (\theta_1 - \theta_2) \quad (12.16)$$

σ —Boltzmann's constant: $\sigma = 5.67 \cdot 10^{-8} \text{ W}/(m^2 K^4)$; ε – emissivity; for a black painted body $\varepsilon = 0.9$; A —radiation area.

In general, for IMs, the radiated energy is much smaller than the energy transferred by convection except for totally enclosed natural ventilation (TENV) or for class F(H) motor with very hot frame (120 to 150°C).

For the case when $\theta_2 = 40^\circ$ and $\theta_1 = 80^\circ C, 90^\circ C, 100^\circ C$, $\varepsilon = 0.9$, $h_{rad} = 7.67, 8.01$, and $8.52 \text{ W}/(m^2 \cdot ^\circ C)$.

For TENV with $h_{co} = 4.56 \text{ W}/(m^2 \cdot ^\circ C)$ (convection) the radiation is superior to convection and thus it cannot be neglected. The total (equivalent) convection coefficient

$$h_{(c+r)0} = h_{co} + h_{rad} \geq 12 \text{ W}/(m^2 \cdot ^\circ C).$$

The convection and radiation combined coefficients $h_{(c+r)0} \approx 14.2 \text{ W}/(m^2 \cdot ^\circ C)$ for steel unsmoothed frames, $h_{(c+r)0} = 16.7 \text{ W}/(m^2 \cdot ^\circ C)$ for steel smoothed frames,

$h_{(c+r)0} = 13.3 \text{ W}/(\text{m}^2, ^\circ\text{C})$ for copper/aluminum or lacquered or impregnated copper windings. In practice, for design purposes, this value of h_{c0} , which enters Equations (12.12 through 12.14), is, in fact, $h_{(c+r)0}$, the combined convection radiation coefficient.

It is well understood that the heat transfer is three dimensional and as K , h_c and h_{rad} are not constants, the heat flow, even under thermal steady state, is a very complex problem. Before advancing to more complex aspects of heat flow, let us work out a simple example.

Example 12.1. One – dimensional simplified heat transfer

In an induction motor with $p_{\text{Co1}} = 500 \text{ W}$, $p_{\text{Co2}} = 400 \text{ W}$, $p_{\text{iron}} = 300 \text{ W}$, the stator slot perimeter $2h_s + b_s = (2.25 + 8) \text{ mm}$, 36 stator slots, stack length: $l_{\text{stack}} = 0.15 \text{ m}$, an external frame diameter $D_e = 0.30 \text{ m}$, finned area frame (4 to 1 area increase by fins), frame length 0.30 m , let us calculate the winding in slots temperature and the frame temperature, if the air temperature increase around the machine is 10°C over the ambient temperature of 30°C and the slot insulation total thickness is 0.8 mm . The ventilator is used and the end connection/coil length is 0.4 .

Solution

First, the temperature differential of the windings in slots has to be calculated. We assume here that all rotor heat losses crosses the airgap and it flows through the stator core toward the stator frame.

In this case, the stator winding in slot temperature differential is (12.3)

$$\Delta\theta_{\text{cos}} = \frac{\Delta_{\text{ins}} p_{\text{Co1}} \left(1 - \frac{l_{\text{endcon}}}{l_{\text{coil}}} \right)}{K_{\text{ins}} N_s (2h_s + b_s) l_{\text{stack}}} = \frac{0.8 \cdot 10^{-3} \cdot 500 \cdot 0.6}{2.0 \cdot 0.36 \cdot 0.058 \cdot 0.15} = 3.83 ^\circ\text{C}$$

Now we consider that stator winding in slot losses, rotor cage losses, and stator core losses produce heat that flows radially through stator core by conduction without temperature differential (infinite conduction!).

Then all these losses are transferred to ambient through the motor frame through combined free convection and radiation.

$$\theta_{\text{core}} - \theta_{\text{air}} = \frac{q_{\text{total}}}{h_{(c+r)0} A_{\text{frame}}} = \frac{(500 + 400 + 300)}{14.2 \cdot \pi \cdot 0.30 \cdot 0.3 \cdot (4/1)} = 74.758 ^\circ\text{C}$$

with $\theta_{\text{air}} = 40^\circ$, $\theta_{\text{ambient}} = 30^\circ$, the frame (core) temperature $\theta_{\text{core}} = 40 + 74.758 = 114.758^\circ\text{C}$ and the winding in slots temperature $\theta_{\text{cos}} = \theta_{\text{core}} + \Delta\theta_{\text{cos}} = 114.758 + 3.83 = 118.58^\circ\text{C}$. In such TENV induction machines, the unventilated stator winding end turns are likely to experience the highest temperature spot. However, it is not at all simple to calculate the end connection temperature distribution.

12.6. HEAT TRANSPORT (THERMAL TRANSIENTS) IN A HOMOGENOUS BODY

Although the IM is not a homogenous body, let us consider the case of a homogenous body – where temperature is the same all over.

The temperature of such a body varies in time if the heat produced inside, by losses in the induction motor, is applied at a certain point in time—as after starting the motor. The heat balance equation is

$$P_{\text{loss}} = M c_t \frac{d(T - T_0)}{dt} + A h_{\text{cond}} (T - T_0) \quad (12.17)$$

losses
per unit
time in W
heat accumulation
in the body
heat transfer from the body
through convection, conduction, radiation

M–body mass (in Kg), c_t –specific heat coefficient (J/(Kg·°C))

A–area of heat transfer from (to) the body

h–heat transfer coefficient

Denoting by

$$C_t = M c_t \quad \text{and} \quad R_{\text{conv}} = \frac{1}{A h} ; \left(R_{\text{cond}} = \frac{1}{K A} \right) \quad (12.18)$$

equation (12.17) becomes

$$P_{\text{loss}} = C_t \frac{d(T - T_0)}{dt} + \frac{(T - T_0)}{R_t} \quad (12.19)$$

This is similar to a R_t , C_t parallel electric circuit fed from a current source P_{loss} with a voltage $T - T_0$ (Figure 12.6).

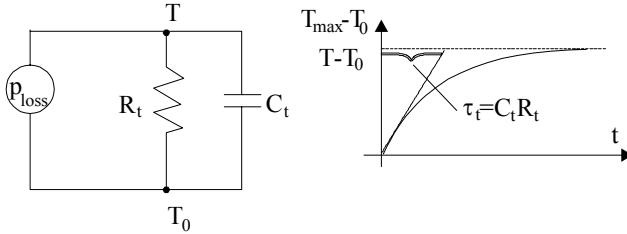


Figure 12.6 Equivalent thermal circuit

For steady state, C_t does not enter Equation (12.17) and the equivalent circuit (Figure 12.6).

The solution of this electric circuit is evident.

$$T = (T_{\text{max}} - T_0) \left(1 - e^{-\frac{t}{\tau_t}} \right) + T_0 e^{-\frac{t}{\tau_t}} \quad (12.20)$$

The thermal time constant $\tau_t = C_t R_t$ is very important as it limits the machine working time with a certain level of losses and given cooling conditions. Intermittent operation, however, allows for more losses (more power) for the same given maximum temperature, $T_{\max}$.

The thermal time constant increases with machine size and effectivity of the cooling system. A TENV motor is expected to have a smaller thermal time constant than a constant speed ventilator-cooled configuration.

12.7. INDUCTION MOTOR THERMAL TRANSIENTS AT STALL

The IM at stall is characterized by very large conductor losses. Core loss may be neglected by comparison. If the motor remains at stall the temperature of the windings and cores increases in time. There is a maximum winding temperature limit $T_{\max}^{\text{copper}}$ given by the insulation class, (155°C for class F) which should not be surpassed. This is to maintain a reasonable working life for conductor insulation. The machine is designed for lower winding temperatures at full continuous load.

To simplify the problem, let us consider two extreme cases, one with long end connection stator winding and the other a long stack and short end connections.

For the first case we may neglect the heat transfer by conduction to the winding in slots portion. Also, if the motor is totally enclosed, the heat transfer through free convection to the air inside the machine is rather small (because this air gets hot easily). In fact, all the heat produced in the end connection (p_{Coend}) serves to increase end winding temperature.

$$\frac{\Delta\theta_{\text{endcon}}}{\Delta t} \approx \frac{p_{\text{Coend}}}{C_{\text{endcon}}}; C_{\text{endcon}} = M_{\text{endcon}} c_{\text{tcopper}} \quad (12.21)$$

with $p_{\text{Coend}} = 1000 \text{ W}$, $M_{\text{endcon}} = 1 \text{ Kg}$, $c_{\text{tcopper}} = 380 \text{ J/Kg}^\circ\text{C}$, the winding would heat up 115°C (from 40 to 155°C) in a time interval Δt .

$$(\Delta t)_{40 \rightarrow 155^\circ\text{C}} = \frac{115 \cdot 1 \cdot 380}{1000} = 43.7 \text{ seconds} \quad (12.22)$$

Now if the machine is already hot at, say, 100°C , $\Delta\theta_{\text{endcon}} = 155^\circ - 100^\circ = 55^\circ\text{C}$. So the time allowed to keep the machine at stall is reduced to

$$(\Delta t)_{100 \rightarrow 155^\circ\text{C}} = \frac{55 \cdot 1 \cdot 380}{1000} = 20.9 \text{ seconds}$$

The equivalent thermal circuit for this oversimplified case is shown on [Figure 12.7a](#).

On the contrary, for long stacks, only the winding losses in slots are considered. However, this time some heat accumulated in the core and the same heat is transferred through thermal conduction through insulation from slot conductors to core.

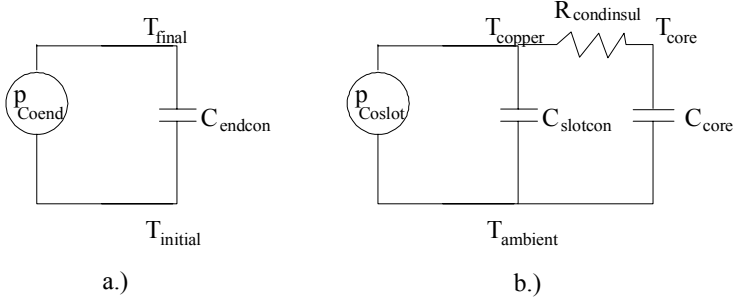


Figure 12.7 Simplified thermal equivalent circuits for stator winding temperature rise at stall
a.) long end connections; b.) long stacks

With $p_{\text{Coslot}} = 1000 \text{ W}$, $M_{\text{slotcopper}} = 1 \text{ Kg}$, $C_{\text{slotcopper}} = 380 \text{ J/Kg}^{\circ}\text{C}$, insulation thickness 0.3 mm $K_{\text{ins}} = 6 \text{ W/m}^{\circ}\text{C}$, $c_{\text{tcore}} = 490 \text{ J/Kg}^{\circ}\text{C}$, slot height $h_s = 20 \text{ mm}$, slot width $b_s = 8 \text{ mm}$, slot number: $N_s = 36$, $M_{\text{core}} = 5 \text{ Kg}$, stack length $l_{\text{stack}} = 0.1 \text{ m}$,

$$R_{\text{condinsul}} = \frac{\Delta_{\text{ins}}}{(2h_s + b_s)l_{\text{stack}}N_sK_{\text{ins}}} = \frac{0.3 \cdot 10^{-3}}{(2 \cdot 20 + 8) \cdot 10^{-3} \cdot 0.1 \cdot 36 \cdot 2} = 8.68 \cdot 10^{-4} (^{\circ}\text{C/m}) \quad (12.23)$$

$$C_{\text{slotcon}} = M_{\text{slotcon}}c_{\text{tcopper}} = 1 \cdot 380 = 380 \text{ J}^{\circ}\text{C} \quad (12.24)$$

$$C_{\text{core}} = M_{\text{core}}c_{\text{tcore}} = 5 \cdot 490 = 2450 \text{ J}^{\circ}\text{C}$$

The temperature rise in the copper and core versus time (solving the circuit of Figure 12.7b) is

$$T_{\text{copper}} - T_{\text{ambient}} = P_{\text{Coslot}} \left[\frac{t}{C_{\text{slotcon}} + C_{\text{core}}} + \frac{\tau_t^2}{C_{\text{slotcon}} \tau_{\text{con}}} \left(1 - e^{-\frac{t}{\tau_t}} \right) \right] \quad (12.25)$$

$$T_{\text{core}} - T_{\text{ambient}} = \frac{P_{\text{Coslot}}}{C_{\text{slotcon}} + C_{\text{core}}} \left[t - \tau_t \left(1 - e^{-\frac{t}{\tau_t}} \right) \right]$$

with

$$\tau_{\text{con}} = C_{\text{slotcon}} R_{\text{condinsul}}; \quad \tau_t = \frac{C_{\text{slotcon}} \cdot C_{\text{core}}}{C_{\text{slotcon}} + C_{\text{core}}} R_{\text{condinsul}} \quad (12.26)$$

As expected, the copper temperature rise is larger than core temperature rise. Also, the core accumulates a good part of the winding-produced heat, so the time after which the conductor insulation temperature limit (155°C for class F) is reached at stall is larger than for the end connection windings.

The thermal time constant τ_t is

$$\tau_t = \frac{380 \cdot 2450}{2450 + 380} \cdot 8.68 \cdot 10^{-4} = 0.2855 \text{ seconds} \quad (11.27)$$

The second term in (12.25) dies out quickly so, in fact, only the first, linear term counts. As $C_{\text{core}} \gg C_{\text{slotcon}}$, the time to reach the winding insulation temperature limit is increased a few times: for $T_{\text{ambient}} = 40^\circ\text{C}$ and $T_{\text{copper}} = 155^\circ\text{C}$ from (12.25).

$$(\Delta t)_{40^\circ\text{C} \rightarrow 155^\circ\text{C}} = \frac{(155 - 40)(380 + 2450)}{1000} = 325.45 \text{ seconds}$$

Consequently, longer stack motors seem advantageous if they are to be used frequently at or near stall at high currents (torques).

12.8. INTERMITTENT OPERATION

Intermittent operation with IMs occurs both in line-start constant frequency and voltage, and in variable speed drives (variable frequency and voltage).

In most line-start applications, as the voltage and frequency stay constant, the magnetization current I_m is constant. Also, the rotor circuit is dominated by the rotor resistance term (R_r/S) and thus the rotor current I_r is 90° ahead of I_m and the torque may be written as

$$T_e \approx 3p_1 L_m I_m I_r = 3p_1 L_m I_m \sqrt{I_s^2 - I_m^2} \quad (12.28)$$

The torque is proportional to the rotor current, and the stator and rotor winding losses and core losses are related to torque by the expression

$$\begin{aligned} P_{\text{dis}} &= p_{\text{core}} + p_{\text{Costator}} + p_{\text{Corotor}} \approx 3R_s I_s^2 + 3R_r I_r^2 + p_{\text{core}} = \\ &= 3R_s \left(I_m^2 + \left(\frac{T_e}{3p_1 L_m I_m} \right)^2 \right) + 3R_r \left(\frac{T_e}{3p_1 L_m I_m} \right)^2 + \frac{3(\omega_1 L_m I_m)^2}{R_{m\parallel}} \end{aligned} \quad (12.29)$$

For fractional power (sub kW) or low speed ($2p_1 = 10, 12$), motors I_m (magnetization current) may reach 70 to 80% of rated current I_{sn} and thus (12.29) remains a rather complicated expression of torque, with $I_n = \text{const}$.

For medium and large power (and $2p_1 = 2, 4, 6$) IMs, in general $I_m < 30\% I_{\text{sn}}$ and I_m may be neglected in (12.29), which becomes

$$P_{\text{dis}} \approx (p_{\text{core}})_{\text{const}} + 3(R_s + R_r) \left(\frac{T_e}{3p_1 L_m I_m} \right)^2 \quad (12.30)$$

Electromagnetic losses are proportional to torque squared. For variable speed drives with IMs, the magnetization current is reduced with torque reduction to cut down (minimize) core and winding losses together.

Thus, (12.29) may be used to obtain $\partial P_{\text{dis}}/\partial I_m = 0$ and obtain $I_m(T_e)$ and, again, from (12.29), $P_{\text{dis}}(T_e)$. Qualitatively for the two cases, the electromagnetic loss variation with torque is shown on Figure 12.8.

As expected, for an on-off sequence (t_{ON} , t_{OFF}), more than rated (continuous duty) losses are acceptable during on time. Therefore, motor overloading is permitted. For constant magnetization current, however, as the losses are proportional to torque squared, the overloading is not very large but still similar to the case of PM motors [3], though magnetization losses $3R_s I_m^2$ are additional for the IM.

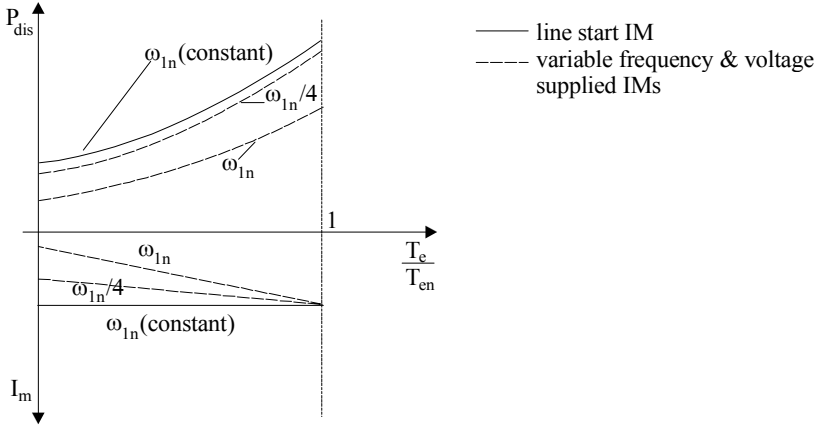


Figure 12.8 Electromagnetic losses P_{dis} and magnetisation current I_m versus torque

The duty cycle d may be defined as

$$d = \frac{t_{\text{ON}}}{t_{\text{ON}} + t_{\text{OFF}}} \quad (12.31)$$

Complete use of the machine in intermittent operation is made if, at the end of ON time, the rated temperature of windings is reached. Evidently the average losses during ON time P_{dis} may surpass the rated losses P_{disn} , for continuous steady state operation. By how much depends both on the t_{ON} value and on the machine equivalent thermal time constant τ_t .

$$\tau_t = R_t C_t \quad (12.32)$$

C_t —thermal capacity of winding ($J/^\circ\text{C}$); R_t —thermal resistance between windings and the surroundings ($^\circ\text{C}/\text{W}$). The value of τ_t depends on machine geometry, rated power and speed, and on the cooling system, and may run from tens of seconds to tens of minutes or even several hours.

12.9. TEMPERATURE RISE (T_{ON}) AND FALL (T_{OFF}) TIMES

The loss (dissipated power) P_{dis} may be considered approximately proportional to load squared.

$$\frac{P_{dis}}{P_{disn}} \approx \left(\frac{T_c}{T_{en}} \right)^2 = K_{load}^2 \quad (12.33)$$

The temperature rise, for an equivalent homogeneous body, during t_{ON} time is (12.19),

$$T - T_0 = RP_{dis} \left(1 - e^{-\frac{t_{ON}}{\tau_t}} \right) + (T_c - T_0) e^{-\frac{t_{ON}}{\tau_t}} \quad (12.34)$$

where T_c is the initial temperature and T_0 the ambient temperature ($T_{max} - T_0 = R \cdot P_{dis}$), with P_{disn} ($K_{load} = 1$), $T(t_{ON}) = T_{rated}$. Replacing in (12.34) P_{dis} by

$$P_{dis} = P_{disn} \cdot K_{load}^2 = \frac{(T_{rated} - T_0)}{R} \cdot K_{load}^2 \quad (12.35)$$

with $T_{max} = T_{rated}$,

$$(T_{rated} - T_0) \left[1 - K_{load}^2 \left(1 - e^{-\frac{t_{ON}}{\tau_t}} \right) \right] = (T_c - T_0) e^{-\frac{t_{ON}}{\tau_t}} \quad (12.36)$$

Equation (12.36) shows the dependence of t_{ON} time, to reach the rated winding temperature, from an initial temperature T_c for a given overload factor K_{load} . As expected t_{ON} time decreases with the rise of initial winding temperature T_c .

During t_{OFF} time, the losses are zero, and the initial temperature is $T_c = T_r$. So with $K_{load} = 0$ and $T_c = T_r$, (12.36) becomes

$$T - T_0 = (T_r - T_0) \cdot e^{-\frac{t_{OFF}}{\tau_t}} \quad (12.37)$$

For steady state intermittent operation, however, the temperature at the end of OFF time is equal, again, to T_c .

$$T_c - T_0 = (T_r - T_0) \cdot e^{-\frac{t_{OFF}}{\tau_t}} \quad (12.38)$$

For given initial (low) T_c , final (high) T_{rated} temperatures, load factor K_{load} , and thermal time constant τ_t , Equations (12.36) and (12.38) allow for the computation of t_{ON} and t_{OFF} times.

Now, introducing the duty cycle $d = \frac{t_{ON}}{t_{ON} + t_{OFF}}$ to eliminate t_{OFF} , from (12.36) and (12.38) we obtain

$$K_{\text{load}} = \sqrt{\frac{1 - e^{-\frac{t_{\text{ON}}}{d\tau_t}}}{1 - e^{-\frac{t_{\text{ON}}}{\tau_t}}}} \quad (12.39)$$

It is to be noted that using (12.36)–(12.38) is most practical when $t_{\text{ON}} < \tau_t$ as it is known that the temperature stabilizes after 3 to $4\tau_t$.

For example, with $t_{\text{ON}} = 0.2 \tau_t$ and $d = 25\%$, $K_{\text{load}} = 1.743$, $\tau_t = \text{minutes}$, it follows that $t_{\text{ON}} = 15 \text{ minutes}$ and $t_{\text{OFF}} = 45 \text{ minutes}$.

For very short on-off cycles ($(t_{\text{ON}} + t_{\text{OFF}}) < 0.2 \tau_t$), we may use Taylor's formula to simplify (12.39) to

$$K_{\text{load}} = \sqrt{\frac{1}{d}} \quad (12.40)$$

For short cycles, when the machine is overloaded as in (12.40), the medium loss will be the rated one.

For a single pulse, we may use $d = 0$ in (12.39) to obtain

$$K_{\text{load}} = \sqrt{\frac{1}{1 - e^{-\frac{t_{\text{ON}}}{\tau_t}}}} \quad (12.41)$$

As expected for one pulse, K_{load} allowed to reach rated temperature for given t_{ON} is larger than for repeated cycles.

With same start and end of the cooling period temperature T_c , the t_{ON} and t_{OFF} times are again obtained from (12.39) and (12.38), respectively, even for a single cycle (heat up, cool down).

$$t_{\text{OFF}} = -\tau_t \ln \left[K_{\text{load}}^2 - (K_{\text{load}}^2 - 1)e^{-\frac{t_{\text{ON}}}{\tau_t}} \right] \quad (12.42)$$

For given K_{load} from (12.41), we may calculate t_{ON}/τ_t , while from (12.42) t_{OFF} time, for a single steady state cycle, T_c to T_r to T_c temperature excursion ($T_r > T_c$) is obtained. It is also feasible to set t_{OFF} and, for given K_{load} , to determine from (12.42), t_{ON} .

Rather simple formulas as presented in this chapter, may serve well in predicting the thermal transients for given overload and intermittent operation.

After this almost oversimplified picture of IM thermal modeling, let us advance one more step by building more realistic thermal equivalent circuits.

12.9 MORE REALISTIC THERMAL EQUIVALENT CIRCUITS FOR IMs

Let us consider the overall heating of the stator (or rotor) winding with radial channels. The air speed and temperatures inside the motor are taken as known. (The ventilator design is a separate problem which, produces the airflow

A half longitudinal cross section is shown in [Figure 12.9a](#) for the stator and in [Figure 12.9b](#) for the rotor.

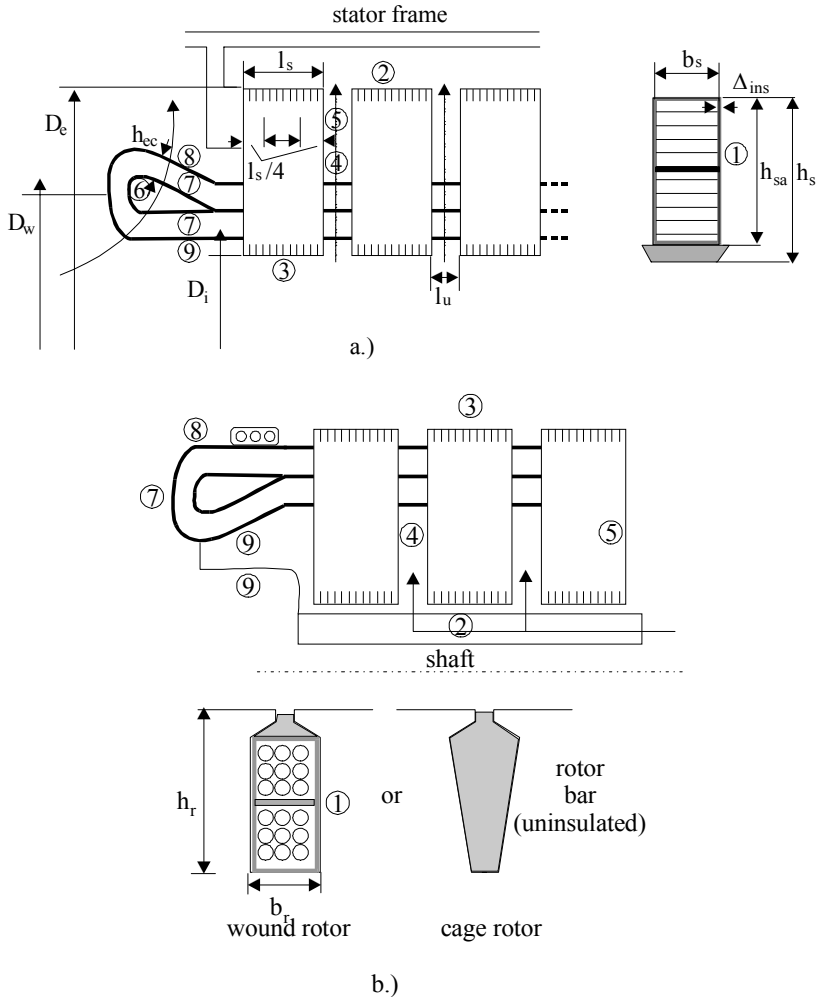


Figure 12.9 IM with radial ventilating channels
a.) stator winding, b.) rotor winding

The objective here is to set a more realistic equivalent thermal circuit and explicitate the various thermal resistances $R_{t1}, \dots, R_{t9}$.

To do so a few assumptions are made.

- The winding end connection losses do not contribute to the stator (rotor) stack heating
- The end-connection and in-slot winding temperature, respectively, do not vary axially or radially
- The core heat center is placed $l_s/4$ away from elementary stack radial channel

The equivalent circuit with thermal resistances is shown in [Figure 12.10](#).

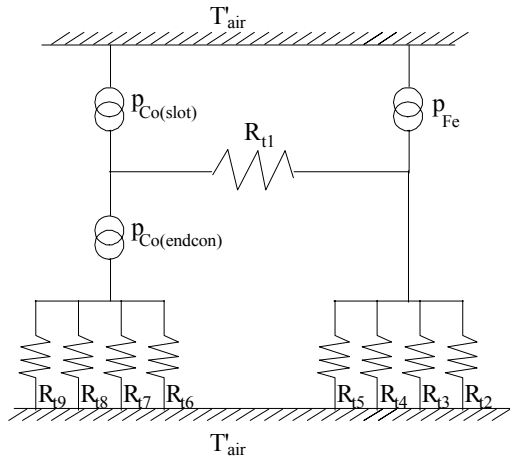


Figure 12.10 Equivalent thermal circuit for stator or rotor windings

T'_{air} – air temperature from the entrance into the machine to the winding surface.

In [Figure 12.10](#),

$P_{Co(slot)}$ – winding losses for the part situated in slots

$P_{Co(endcon)}$ – end connection winding losses

P_{Fe} – iron losses

R_{t1} – slot insulation thermal resistance from windings in slots to core

R_{t2} – thermal resistance from core to air which is exterior (interior) to it – stator core to frame; rotor core to shaft

R_{t3} – thermal resistance from core to airgap cooling air

R_{t4} – thermal resistance from the iron core to the air in the ventilation channels

R_{t5} – thermal resistance from core to air in ventilation channel

R_{t6} – thermal resistance towards the air inside the end connections (it is ∞ for round conductor coils)

R_{t7} – thermal resistance from the frontal side of end connections to the air between neighbouring coils

R_{t8} – thermal resistance from end connections to the air above them

R_{t9} – thermal resistance from end connections to the air below them

Approximate formulas for $R_{t1} - R_{t9}$ are

$$R_{t1} = \frac{\Delta_{ins}}{K_{ins}A_1}; \quad A_1 \approx (2h_s + b_s)n_s l_s N_s \quad (12.43)$$

n_s – number of elementary stacks ($n_s - 1$ – radial channels)

l_s – elementary stack length; N_s – stator slot number

K_{ins} – slot insulation heat transfer coefficient

$$R_{t2} = \frac{h_{cs(r)}}{K_{Fe}A_2} + \frac{1}{h_{c2}A_2}; \quad A_2 = (\pi D_e - b_f n_f)n_s l_s \quad (12.44)$$

b_f – fins width, n_f – fins number for the stator. For the rotor, $b_f = 0$ and D_e is replaced by rotor core interior diameter, $h_{cs(r)}$ – back core radial thickness, K_{core} – core radial thermal conductivity and h_{c2} – thermal convection coefficient (all parameters in IS units). Note that h_{c3} is influenced by the air speed as in (12.14).

$$R_{t3} = \frac{h_{sa}}{K_{Fe}A_3} + \frac{1}{h_{c3}A_3}; \quad A_3 = (\pi D_i - b_f N_s)n_s l_s \quad (12.45)$$

h_{c3} is the convection thermal coefficient as influenced by the air speed in the airgap in presence of radial channels (Equation 12.14).

$$R_{t4} = \frac{\Delta_{inscon}}{K_{copper}A_4} + \frac{1}{h_{c4}A_4}; \quad A_4 = N_s(2h_{s0} + b_s)(n_s - 1)l_v \quad (12.46)$$

l_v – axial length of ventilation channel

$$R_{t5} = \frac{2(n_s - 1)(l_s / 4)}{K_{Felong}A_{5s(r)}} + \frac{1}{h_{c5}A_{5s(r)}} \quad (12.47)$$

$$A_{5s} = 2n_s \left(\frac{\pi}{4} (D_e^2 - (D_i + 2h_s)^2) + N_s b_{ts} h_s \right)$$

$$A_{5r} = 2n_s \left(\frac{\pi}{4} ((D_i - 2h_r)^2 - D_{ir}^2) + N_r b_{tr} h_s \right) \quad (12.48)$$

N_r – rotor slots, $b_{ts(r)}$ – stator (rotor) tooth width, h_{c5} – convection thermal coefficient as influenced by the air speed in the radial channels.

$$R_{t6} = \frac{\Delta_{inscon}}{K_{copper}A_6} + \frac{1}{h_{c6}A_6}$$

$$R_{t7} = \frac{\Delta_{inscon}}{K_{copper}A_7} + \frac{1}{h_{c7}A_7}$$

$$R_{t8} = \frac{\Delta_{\text{inscon}}}{K_{\text{copper}} A_8} + \frac{1}{h_{c8} A_8} \quad (12.49)$$

$$R_{t9} = \frac{\Delta_{\text{inscon}}}{K_{\text{copper}} A_9} + \frac{1}{h_{c9} A_9}$$

$R_{t6} - R_{t9}$ refer to winding end connection heat transfer by thermal conduction through the electrical insulation and, by convention, through the circulating air in the machine. Areas of heat transfer $A_6 - A_9$ depend heavily on the coils shape and their arrangement as end connections in the stator (or rotor).

For round wire coils with insulation between phases, the situation is even more complicated as the heat flow through the end connections toward their interior or circumferentially may be neglected ($R_6 = R_7 = \infty$).

As the air temperature inside the machine was considered uniform, the stator and rotor equivalent thermal circuits as in [Figure 12.10](#) may be treated rather independently ($p_{Fe} = 0$ in the rotor, in general). In the case where there is one stack (no radial channels), the above expressions are still valid with $n_s = 1$ and, thus, all heat transfer resistances related to radial channels are ∞ ($R_4 = R_5 = \infty$).

12.10. A DETAILED THERMAL EQUIVALENT CIRCUIT FOR TRANSIENTS

The ultimate detailed thermal equivalent circuit of the IM should account for the three dimensional character of heat flow in the machine.

Although this may be done, a two dimensional model is used. However we may break the motor axially into a few segments and “thermally” connect these segments together.

To account for thermal transients, the thermal equivalent circuit should contain thermal resistances $R_{ti}(^{\circ}\text{C}/\text{W})$ and capacitors $C_{ti}(\text{J}/^{\circ}\text{C})$ and heat sources (W) ([Figure 12.11](#)).

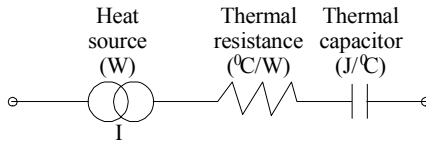


Figure 12.11 Thermal circuit elements with units

A detailed thermal equivalent circuit—in the radial plane—emerges from the more realistic thermal circuit of [Figure 12.9](#) by dividing the heat sources into more components ([Figure 12.12](#)).

The stator conductor losses are divided into their in-slot and overhang (end-connection) components. The same thing could be done for the rotor (especially for wound rotors). Also, no heat transport through conduction from end connections to the coils section in slot is considered in [Figure 12.12](#), as the axial heat flow is neglected.

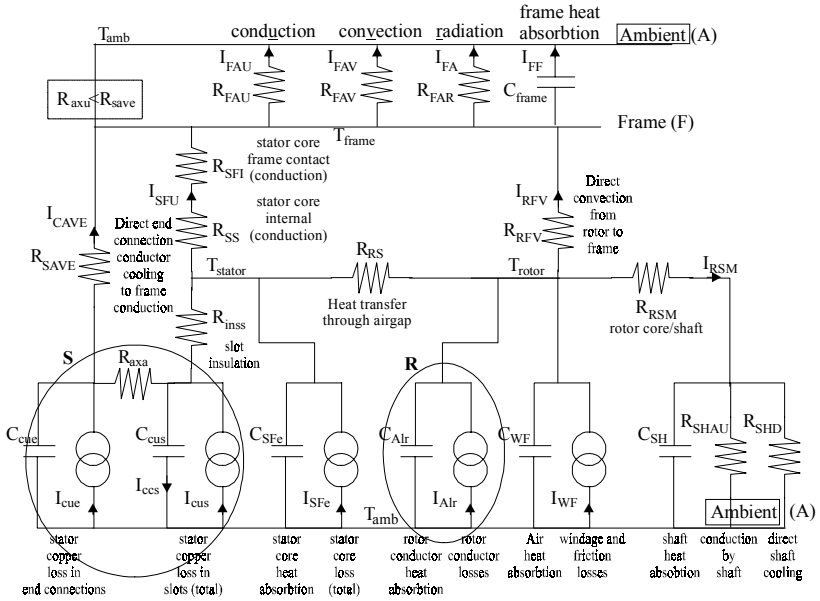


Figure 12.12 A detailed thermal equivalent circuit for IMs

Due to the machine pole symmetry, the model in Figure 12.12 is in fact one dimensional, that is the temperatures vary only radially. A kind of similar model is to be found in Reference [3] for PM brushless motors and in Reference 4 for induction motors.

In Reference [5], a thermal model for three-dimensional heat flow is presented.

It is possible to augment the model with heat transfer along circumferential direction and along axial dimension to obtain a rather complete thermal equivalent circuit with hundreds of nodes.

12.11. THERMAL EQUIVALENT CIRCUIT IDENTIFICATION

As shown in paragraph 12.9, the various thermal resistances R_{ti} (or conductances $G_{ti} = 1/R_{ti}$) and thermal capacitances (C_{ti}) may be approximately calculated through analytical formulas.

As the thermal conductivities, K_i , convection and radiation heat transfer coefficients h_i are dependent on various geometrical factors, cooling system and average local temperature, and at least for thermal transients, their identification through tests would be beneficial.

Once the thermal equivalent circuit structure is settled (Figure 12.12, for example), with various temperatures as unknowns, its state-space matrix equation system is

$$\frac{dX}{dt} = AX + BP \quad (12.50)$$

$$\begin{aligned} X &= [T_1 \dots T_j \dots T_n]^T \text{ - temperature matrix} \\ P &= [P_1 \dots P_j \dots P_m]^T \text{ - power loss matrix} \end{aligned} \quad (12.51)$$

P_j have to be known from the electromagnetic model. A and B are coefficient matrixes built with R_{ti} , C_{ti} .

Ideally n temperature sensors to measure $T_1, \dots, T_n$ versus time would be needed. If it is not feasible to install so many, the model is to be simplified so that all temperatures in the model are measured.

Having experimental values of $T_i(t)$, system (12.50) may be used to determine, by an optimization method, the parameters of the equivalent circuit. In essence, the squared error between calculated and measured (after filtering) temperatures is to be minimum over the entire time span. In Reference 6 such a method is used and the results look good.

As some of the thermal parameters may be calculated, the method can be used to identify them from the losses and then check the heat division from its center.

For example, it may be found that for low power IMs at rated speed, 65% of the rotor cage losses is evacuated through airgap, 20% to the internal frame, and 15% by shaft bearing.

Also the heat produced in the stator end connection windings for such motors is divided as: 20% to the internal frame and end shields (brackets) by convection and the rest of 80% to the stator core by conduction (axially).

Consequently, based on such results, the detailed equivalent circuit should contain a conduction resistance branch from end connections to stator core as 80% of the heat goes through it (R_{axa} in [Figure 12.12](#)).

As an example of an ingenious procedure to measure the R_i , C_i parameters, or the loss distribution, we notice here the case of turning off the IM and measuring the temperature decrease in location of interest versus time.

From (12.18) in the steady state conditions:

$$p_{\text{loss}} = \frac{1}{R_t} (T_0 - T_a) \quad (12.52)$$

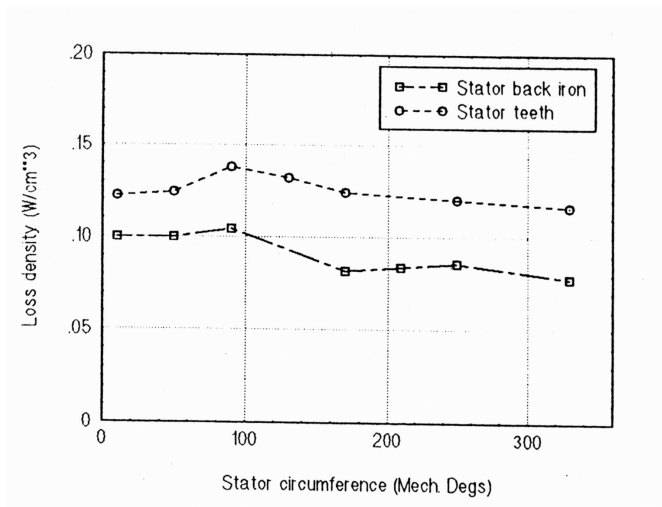
The temperature derivative at $t = 0$ (from 12.18), when the heat input is turned off, is

$$\left(\frac{dT}{dt} \right)_{t=0} = -\frac{1}{R_t C_t} (T_0 - T_a) \quad (12.53)$$

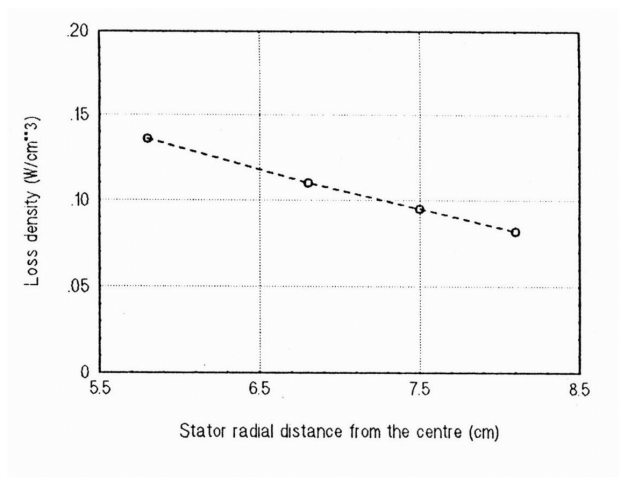
Finally

$$p_{\text{loss}} = -C_t \left(\frac{dT}{dt} \right)_{t \rightarrow 0} \quad (12.54)$$

Measuring the temperature gradient at the moment when the motor is turned off, with C_t known, allows for the calculation of local power loss in the machine just before the machine was turned off. [7,8]



a.)



b.)

Figure 12.13 Iron loss density distribution (W/cm^3)

a.) along circumferential direction, b.) along radial direction, c.) along axial direction
(continued)

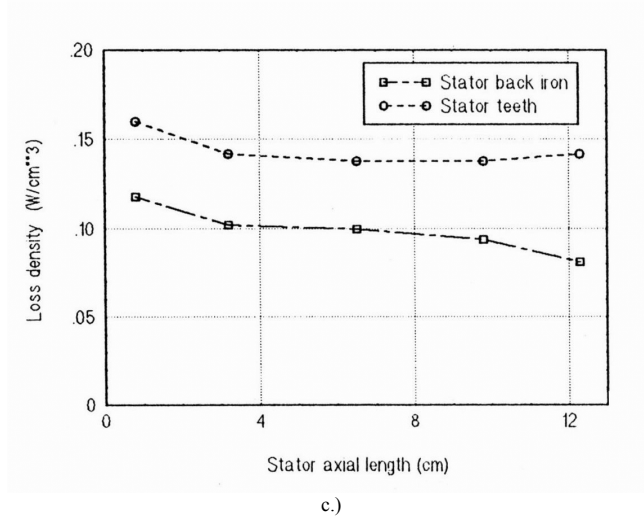


Figure 12.13 (continued)

This way the radial or axial variation of losses (especially in the core) may be obtained, provided that small enough temperature sensors are placed in key locations. The winding average temperature is measured after turn-off by the d.c. voltage/current method of stator winding resistance.

$$R_s(T) = (R_s)_{20^{\circ}\text{C}} \left[1 + \frac{1}{273} (T - T_{\text{amb}}) \right] \quad (12.55)$$

Typical results are shown in [Figure 12.13](#). [8] For the 4 pole IM in question, the temperature variation along circumferential and axial directions in the back core is small, but it is notable in the stator teeth. A notable decrease of loss density with radial distance is present, as expected, ([Figure 12.13c](#)).

12.12. THERMAL ANALYSIS THROUGH FEM

In theory, the three-dimensional FEM alone could lead to a fully realistic temperature distribution for a machine of any power and size, provided the localization of heat sources and their levels are known.

The differential equation for heat flow is

$$\nabla(K\Delta T) + p_{\text{loss}} = \gamma c \frac{\partial T}{\partial t} \quad (12.56)$$

with K – local thermal conductivity ($\text{W/m}^{\circ}\text{C}$); γ – local density (Kg/m^3), p_{loss} – losses per unit volume (W/m^3); and c – specific heat coefficient ($\text{J}^{\circ}\text{CKg}$). The coefficients K , γ , c vary throughout the machine.

Two types of boundary conditions are usually present:

- Dirichlet conditions:

$$T(x_b, y_b, z_b, t) = T^* \quad (12.57)$$

The ambient temperature is such a boundary around the machine.

- Newman conditions:

$$q - K_x \frac{\partial T}{\partial x} n_x - K_y \frac{\partial T}{\partial y} n_y - K_z \frac{\partial T}{\partial z} n_z = 0 \quad (12.58)$$

n_x, n_y, n_z are the x, y, z components of unit vector rectangular to the respective boundary surface; q – the heat flow through the surface.

As a 3D FEM would require large amounts of computation time, 2D FEM models have been built to study the temperature distribution either in the radial cross section or in the axial cross section.

The radial cross section has a geometrical symmetry as seen in [Figure 12.14](#).

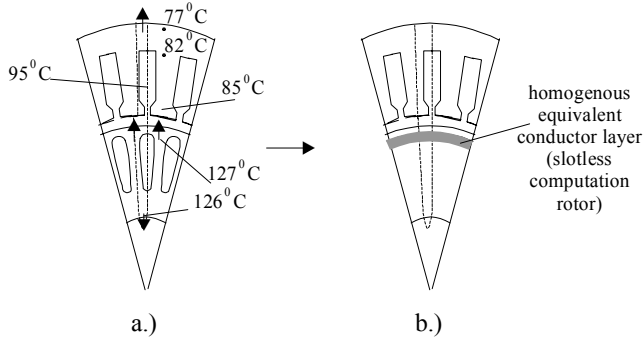


Figure 12.14 Computation sector a.) and its restructuring to avoid motion influence b.)

To avoid the rotation influence on the modeling in the airgap zone, the rotor sector is replaced by a motion-independent computation sector ([Figure 12.14b](#)). [9]

After the temperature on the rotor surface is calculated and considered “frozen,” the actual rotor sector is modeled.

It was found that the stator temperature varies radially and axially in a visible manner, while the temperature gradient is much smaller in the rotor ([Figure 12.14a](#)). [9] Similar results with 2D FEM are presented in Reference [10].

3D FEM may be used for a more complete IM thermal modeling, treated independently or together with the electromagnetic model for load (or speed) transients; however the computation time becomes large.

Still, due to the dependence of thermal resistances and capacitors and loss distribution on many “imponderable” factors, experimental methods (such as advanced calorimetric ones) are needed to validate theoretical results, especially when new machine configurations or design specifications are encountered.

12.13. SUMMARY

- Any IM design is thermally limited according to the conductor insulation class mainly: A(105°C), B(130°C), F(155°C), H(180°C)
- The insulation life decreases very rapidly with temperature increase over the rated value (for extra 10°C the life may be halved).
- Cooling methods, used to extract the heat from the machine and thus limit the temperature, are paramount in IM design.
- From totally enclosed, naturally ventilated (TENV) induction motors through drip-proof axial internal, radial internal to radial-axial cooling, the sophistication of cooling methods and their heat removal capacity increases steadily.
- Heat transfer takes place through thermal conduction, convection, and radiation. For induction machine design, convection and radiation heat transfer coefficients are usually lumped together. Radiation is notable for TENV configurations.
- The heat transfer steady state and transients may be approached either by equivalent thermal circuits (similar to R–C electric circuits) or by numerical methods (FEM).
- Thermal resistances R_{ti} (°C/W) are introduced for conduction, convection, and radiation heat transfer and thermal capacitances C_{ti} (J/°C) stand for heat absorption in the various parts of the motor.
- The heat sources p_{lossi} (W) represent current sources in the equivalent circuit.
- Thermal resistances decrease with cooling air speed.
- The windings have the highest temperature spots, in general.
- It has been shown that, at least in small power IMs, about 80% of the heat (loss) in the stator coils overhangs is transmitted through conduction to the winding part in slots and to the core.
- It has also been shown that 60 to 70% of rotor losses are transferred through the airgap to the stator core.
- The axial variation of temperature depends on the relative stack length, method of cooling, and machine power. However, for powers up to hundreds of kW, unistack motors with axial external cooling and finned frames have recently become popular for general purpose designs.
- Information like this should be instrumental into developing adequate simplified equivalent thermal circuits. FEM–2D and 3D, are very instrumental in getting information for designing practical equivalent circuits.
- Due to the thermal modeling complexity, theory and tests should go hand in hand in the future.

12.14. REFERENCES

1. S.A. Nasar, Handbook of Electric Machines, McGraw-Hill Inc., Chapter 12 (by A.J.Spisak).
2. W.H. McAdams, Heat Transmission, McGraw-Hill, New York, 1942, 2nd edition.
3. J. Hondershot, T.J.E. Miller, PM Brushless Motor Designs, OUP, 1995, Chapter 15.
4. G. Bellenda, L. Ferraris, A. Tenconi, A New Simplified Thermal Model for Induction Motors for EVs Applications, Record of 7th International Conference on EMD, IEE Conf, Publ. 412, 195, Durham, UK, pp.11 – 15.
5. W. Guyglewics – Kacerka, J. Mukosiej, Thermal Analysis of Induction Motor with Axial and Radial Cooling Ducts, Record of ICEM – 1992, Vol.3., pp.971 – 975.
6. G. Champenois, D. Roye, D.S. Zhu, Electrical and Thermal Performance Predictions in Inverter-fed Squirrel Cage Induction Motor Drives, EMPS – Vol.22, No.3, 1994, pp.355 – 369.
7. M. Benamrouche et al., Determination of Iron Loss Distribution in Inverter-fed Induction Motors, EMPS Vol.25, No.6, 1997, pp.649 – 660.
8. H. Benamrouche et.al., Determination of Iron and Stray Load Losses in Induction Motors Using a Thermometric Method, EMPS Vol.26, No.1, 1998, pp.3 – 12.
9. J. Roger, G. Jinenez, The Finite Element Application to the Study of the Temperature Distribution Inside Electric Rotating Machines, Record of ICEM – 1992, Vol.3., pp.976 – 980.
10. D. Sarkar, Approximate Analysis of Temperature Rise in an Induction Motor During Dynamic Braking, EMPS Vol.26, No.6, 1998, p.585 – 599.